

10.1 Inverse Functions

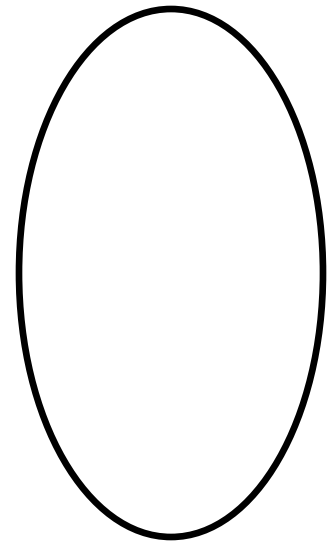
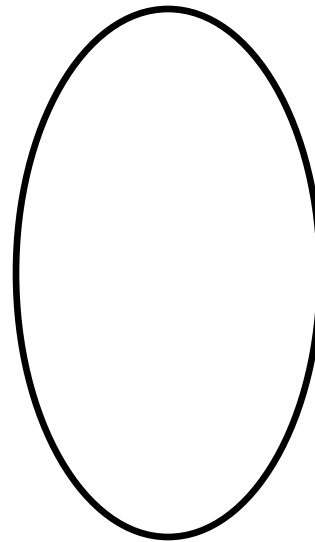
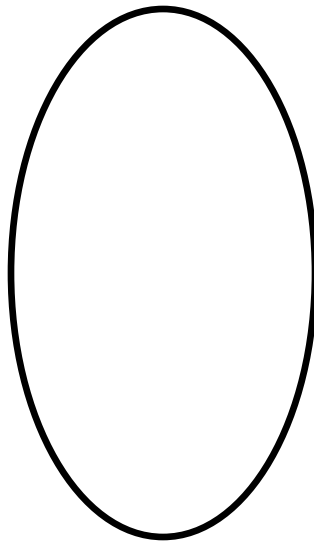
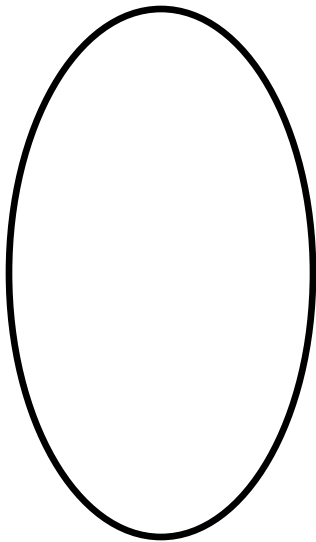
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Objectives:

- Decide whether a function is one-to-one and, if it is, find its inverse.
- Use the horizontal line test to determine whether a function is one-to-one.
- Find the equation of the inverse of a function.
- Graph f^{-1} given the graph of f .

Review of Functions:

- A **function** is a relation in which, for each value of the first component of the ordered pairs, there is **exactly one value** of the second component.



One-to-One and Inverse Functions

One-to-One Function

In a one-to-one function, each x -value corresponds to only one y -value, and each y -value corresponds to only one x -value.

Inverse of a Function

The inverse of a one-to-one function f , written f^{-1} , is the set of all ordered pairs of the form (y, x) , where (x, y) belongs to f . Since the inverse is formed by interchanging x and y , the domain of f becomes the range of f^{-1} and the range of f becomes the domain of f^{-1} .



Finding Inverses of One-to-One Functions

Decide whether each function is one-to-one. If it is, find the inverse.

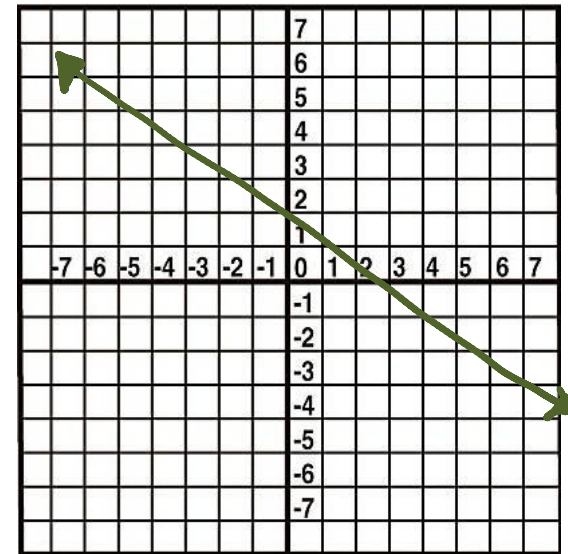
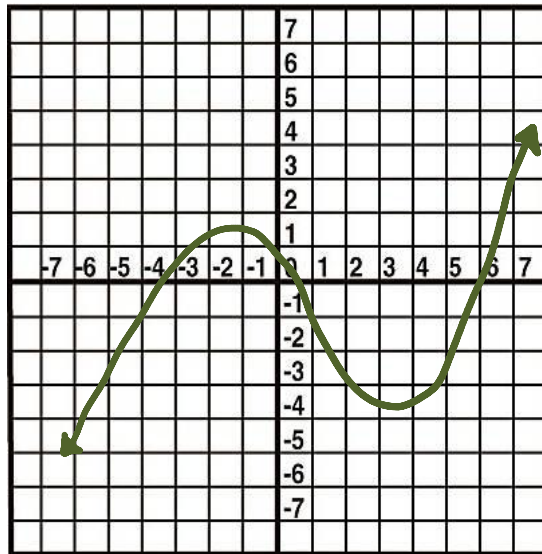
- $F = \{(2, 5), (3, 6), (4, 8), (8, 7)\}$

- $G = \{(0, 3), (-1, 2), (1, 3)\}$

Using the Horizontal Line Test

Horizontal Line Test

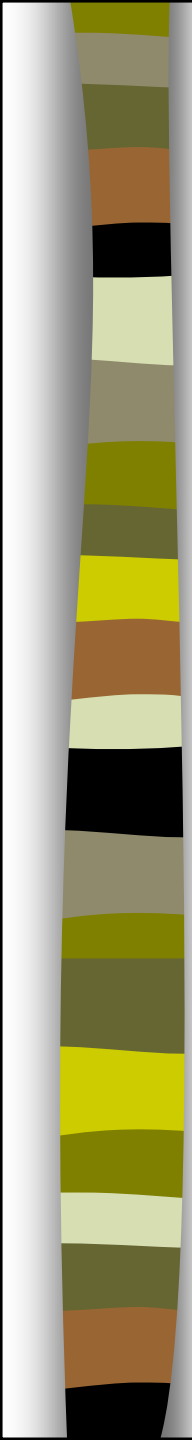
A function is one-to-one if every horizontal line intersects the graph of the function at most once.



Finding the Equation of the Inverse Function

- For a one-to-one function f defined by an equation $y = f(x)$, find the defining equation of the inverse as follows
 - Step 1: Interchange x and y .
 - Step 2: Solve for y .
 - Step 3: Replace y with $f^{-1}(x)$.

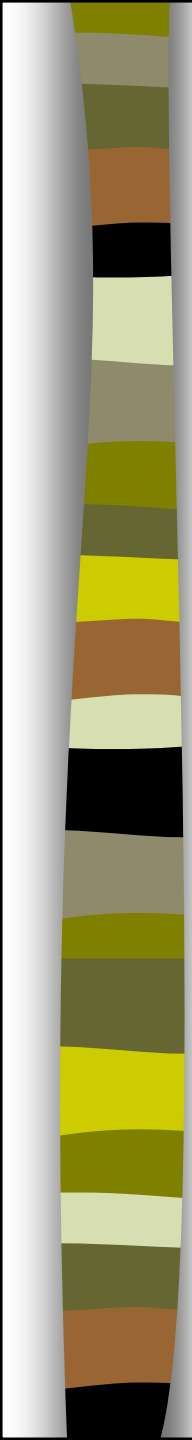
$$f(x) = 2x + 5$$



Finding the Equation of the Inverse Function

- Decide whether each equation defines a one-to-one function. If so, find the equation that defines the inverse.

$$f(x) = 3x - 4$$



Finding the Equation of the Inverse Function

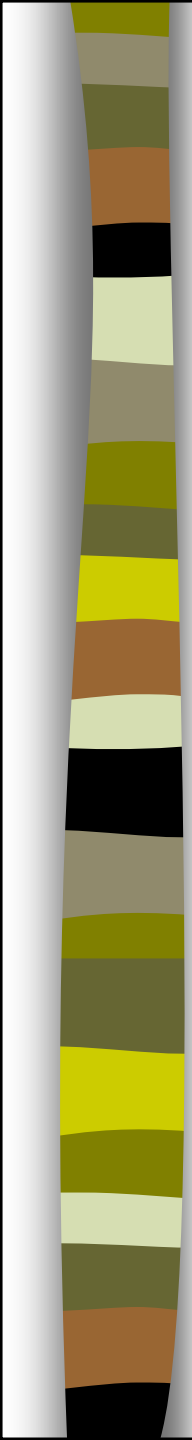
- Decide whether each equation defines a one-to-one function. If so, find the equation that defines the inverse.

$$f(x) = (x - 3)^2$$

Finding the Equation of the Inverse Function

- Decide whether each equation defines a one-to-one function. If so, find the equation that defines the inverse.

$$f(x) = \sqrt{x - 3}, \quad x \geq 3$$



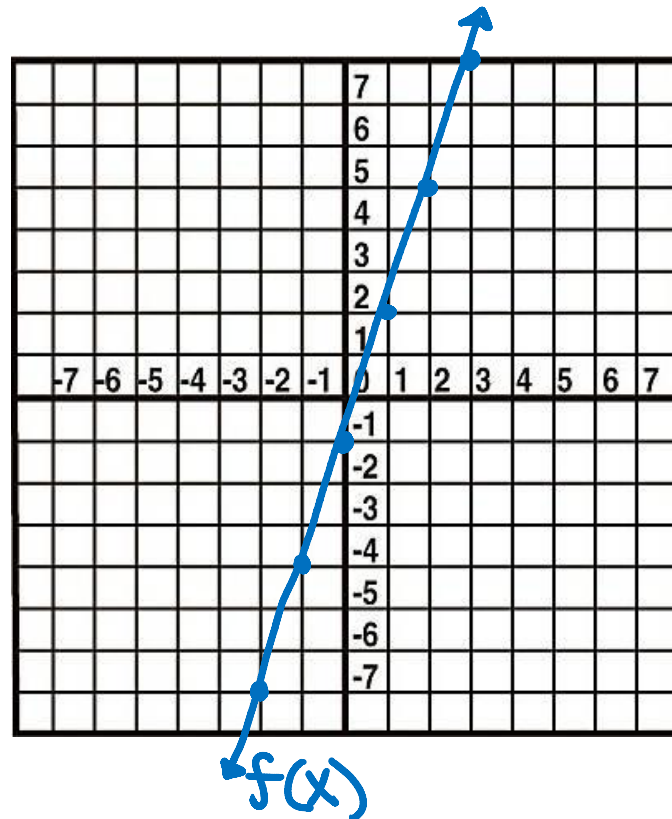
Finding the Equation of the Inverse Function

- Decide whether each equation defines a one-to-one function. If so, find the equation that defines the inverse.

$$f(x) = x^3 + 1$$

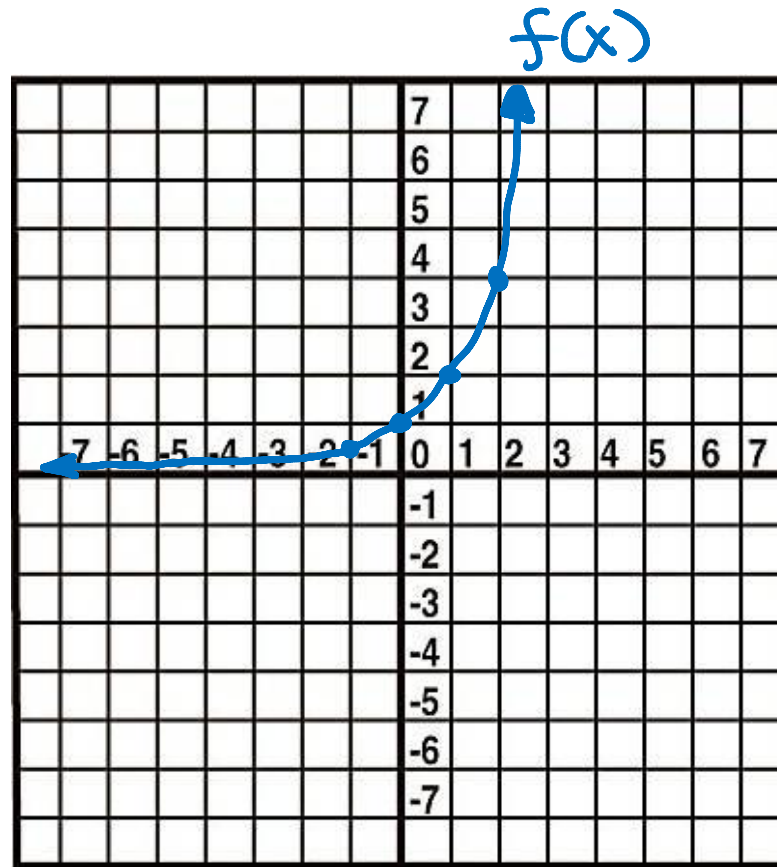
Graphing the Inverse

- Find several ordered pairs that belong to f .
- Interchange x and y to obtain ordered pairs that belong to f^{-1} .
- Plot those points, and sketch the graph of f^{-1} through them.



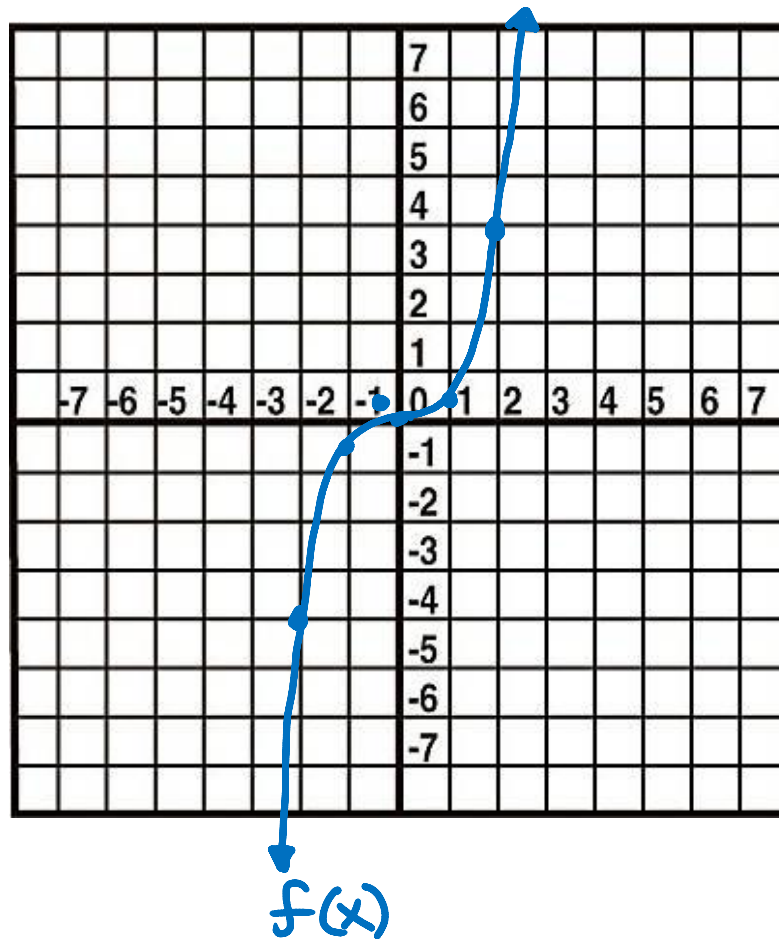
Graphing the Inverse

- Graph the inverse of the function f (shown in blue).



Graphing the Inverse

- Graph the inverse of the function f (shown in blue).





Finding the Value of a Function and It's Inverse

- The function $f(x) = 3^x$ is a one-to-one function.
Find $f(4)$ and $f^{-1}(81)$.



Finding the Value of a Function and It's Inverse

- The function $f(x) = \left(\frac{1}{2}\right)^x$ is a one-to-one function.
Find $f(-3)$ and $f^{-1}(8)$.